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3. It is worthy of remark that Hesse's general theorem enunciated in (1) leads directly to another of at least equal importance, viz. *Any power of a symmetrical determinant is itself expressible as a symmetrical determinant.* For, the *first* power being symmetrical, the *third* must be so also; therefore also the *fifth*, the *seventh*, &c.; and as for the *even* powers, they are known to be symmetrical in the case of any determinant. In the order of discovery, however, this theorem preceded Hesse's, being used by Sylvester in the statement of the property found by him to belong to the equation of the secular inequalities of the planets. In this connection no proof was given of it except in the *Nouvelles Annales*, the editor of which, in bringing the property referred to before his readers, prefaced the statement of the same by a few remarks on determinants. To him apparently the responsibility attaches of discovering the theorem "Le produit de déterminants symétriques est un déterminant symétrique," in order that the accustomed easy step might be made from *product* to *power*.

BEECHCROFT, BISHOPTON, SCOTLAND, *August* 1st, 1881.

IV.

On Newton's Method of Approximation.

BY F. FRANKLIN.

Let $f(x) = 0$ be an algebraic equation. Newton's method of approximation consists in adding to an approximate value, a, of a root, the correction k, $= -\dfrac{f(a)}{f'(a)}$; correcting the new value $a + k$, say a_1, in like manner, viz. by adding $-\dfrac{f(a_1)}{f'(a_1)}$; and so on. Fourier's theorem concerning this method is as follows: If between a and b there is one and only one root of the equation $f(x) = 0$, and *if neither* $f'(x)$ *nor* $f''(x)$ *vanishes* between these limits, then we will be sure to approximate indefinitely to the root by Newton's method, if we begin the process at that one of the quantities a, b for which f has the same sign as f''.

The proof, as usually given, is somewhat tedious; the following proof is very brief, shows that a part of the usual statement of the theorem should be omitted, and gives immediately a measure of the rapidity of the approximation.

The Newtonian correction, k, is $-\frac{f(a)}{f'(a)}$. Denote the true correction by h, so that $a+h$ is the root. Then

$$0=f(a+h)=f(a)+hf'(a)+\frac{1}{2}h^2f''(a+\theta h),$$

whence
$$h=-\frac{f(a)}{f'(a)}-\frac{1}{2}h^2\frac{f''(a+\theta h)}{f'(a)},$$

so that k has the same sign as h and is less in absolute value than h, provided $f''(a+\theta h)$ has the same sign as $f(a)$. That is, the corrected value $a+k$ (say a_1) will be nearer to the root than a and on the same side of the root as a, provided that f'' has, throughout the interval in question, the sign of $f(a)$; and since, if this condition is fulfilled, $f(a)$ and $f(a_1)$ have like signs, the same condition will be fulfilled for a_1. Thus the theorem above stated is proved, with the substitution, for the words in italics, of the words *if* $f''(x)$ *does not change sign.* That is, only one condition is required, the one relating to the *first* derived function being superfluous: the geometrical meaning of this fact is obvious.

The error after the first correction is

$$-\frac{1}{2}h^2\frac{f''(a+\theta h)}{f'(a)},$$

i. e. it cannot exceed in absolute value the product of half the square of the original error by $\frac{F''}{f'(a)}$, where F'' denotes the numerically greatest value of $f''(x)$ between $x=a$ and $x=b$.

Serret, in his *Cours d'Algèbre Supérieure* (vol. 2, pp. 346–348), deduces by a long and somewhat difficult method the result that the error after the first approximation cannot exceed the product of $\frac{1}{2}h^2$ by the numerically greatest value of the fraction $\frac{f''(x)}{f'(x)}$ between $x=a$ and $x=b$. This is obviously less accurate than the result obtained above; and in fact it may be noted that $f'(a)$ is necessarily the numerically *greatest* value which the denominator, $f'(x)$, can take between the limits.

BALTIMORE, *May*, 1881.